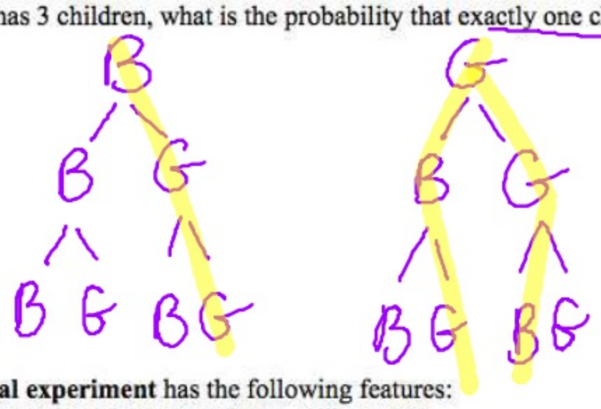
FST 10.5 Notes
Topic: Binomial Probabilities
GOAL
This lesson shows why ${}_nC_k p^k q^{1-k}$ works and applies it to a variety of situations.

SPUR Objectives
H Determine probabilities in situations involving binomial experiments.

Vocabulary
binomial experiment

binomial experiment An experiment with a fixed number of independent trials, each with only two possible outcomes, often called success and failure, and each with the same probability of success.

Warm up
A family has 3 children, what is the probability that exactly one child is a boy?



$\frac{3}{8} = .375$

A **binomial experiment** has the following features:

1) There are repeated situations, called *trials*.
2) There are a fixed number of trials.
3) For each trial, there are only two possible outcomes, often called *success* (S) and *failure* (F).
4) The probability of success is the same in each trial.
5) The trials are independent events.

Binomial Probability Theorem

Suppose that in a binomial experiment with n trials, the probability of success is p in each trial and the probability of failure is q, where $q = 1 - p$. Then

$$P(\text{exactly } k \text{ successes}) = {}_nC_k \cdot p^k q^{n-k} = \binom{n}{k} p^k q^{n-k}.$$

Binomial Probability Formula (another version of the formula)

$$P = {}_nC_r p^r (1-p)^{n-r}$$

n = # of trials
r = successes
little p = probability of success
1-p = probability of failure

Warm up (Solve using the Binomial Probability Formula)
A family has 3 children, what is the probability that exactly one child is a boy?

boy $= \frac{1}{2} = .5$
girl $= \frac{1}{2} = .5$

$${}_3C_1 (.5)^1 (1-.5)^{3-1}$$

$${}_3C_1 (.5)^1 (.5)^2 = \boxed{.375 = 37.5\%}$$

Example 1
Some hereditary diseases are inherited by one-forth of the offspring of the families in which the hereditary gene is present. If such a family has four offspring, what is the probability that exactly one of the offspring inherits the gene?

	T	t
T	TT	Tt
t	Tt	tt

disease $= \frac{1}{4} = .25$
Not disease $= \frac{3}{4} = .75$

$${}_4C_1 (.25)^1 (.75)^3 = \boxed{.422 = 42.2\%}$$

Example 2
Suppose you feel that you have a 90% probability of correctly answering any question on an upcoming history test. If there are ten questions on the test, what is the probability that you will correctly answer 80% or more of the questions ?

$.8(10) = 8$

8 or more

Correct $= .90$
Not correct $= .10$

$${}_{10}C_8 (.90)^8 (.10)^2 + {}_{10}C_9 (.90)^9 (.10)^1 + {}_{10}C_{10} (.90)^{10} (.10)^0 = \boxed{.9298}$$

$= 93\%$ 92.98%